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ELECTROSTATICS

Charge

Charge is the inherent property of certain fundamental particles. It accompanies them wherever they exist. Commonly known charged particles are protons and electrons. The charge of a proton is taken as positive and that of an electron is taken as negative. It is represented by symbol e.

$$e = 1.6 \times 10^{-19} \text{ coulomb}$$

Positive and negative signs were arbitrarily assigned by Benjamin Franklin. This does not mean that the charge of a proton is greater than the charge of an electron.

Quantization of Charge

Electric charges appear only in discrete amounts; it is said to be quantized.

$$Q = \pm ne$$

Where n is an integer.

Conservation of Charge

For an isolated system, the total charge remains constant. Charge is neither created nor destroyed; it is transferred from one body to another.

Coulomb's Law

The force of interaction between two stationary point charges in a vacuum is directly proportional to the product of these charges and inversely proportional to the square of their separation.

$$F = \frac{kq_1q_2}{r^2}$$

Where k is a constant that depends on the system of units. Its value in SI unit is:

$$k = 9 \times 10^9 \text{ Nm}^2\text{C}^{-2}$$

The constant is often written in the form:

$$k = \frac{1}{4\pi\epsilon_0}$$

Where ϵ_0 is called the permittivity constant, numerically equal to 8.85×10^{-12} $\text{C}^2\text{N}^{-1}\text{m}^{-2}$.

Electric Field

The region around a charge where its influence can be felt by another charge.

- **Electric Field Intensity ($\vec{E}$):** The force experienced by a unit positive test charge placed at that point.

$$\vec{E} = \frac{\vec{F}}{q_0}$$

  Its SI unit is N/C or V/m.

- **Electric Field due to a Point Charge:**

$$E = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}$$

Electric Lines of Force

Imaginary lines representing the direction of the electric field.

- Originate from positive charges and terminate on negative charges.
- Never intersect each other.
- Are perpendicular to the surface of conductors.
- The density of lines indicates field strength.

Electric Potential

The amount of work done to bring a unit positive test charge from infinity to a point in an electric field without acceleration.

$$V = \frac{W}{q_0}$$

Its SI unit is Volt (V) or J/C.

- **Electric Potential due to a Point Charge:**

$$V = \frac{1}{4\pi\epsilon_0}\frac{q}{r}$$

- **Potential Difference (ΔV):** Work done per unit charge to move a charge between two points.

$$\Delta V = V_B - V_A = \frac{W_{AB}}{q_0}$$

Electric Potential Energy

The work done in bringing a charge from infinity to a point in an electric field.

$$U = W = qV$$

- **Potential Energy of a System of Two Charges:**

$$U = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r}$$

Electric Dipole

A pair of equal and opposite charges separated by a small distance.

- **Electric Dipole Moment ($\vec{p}$):**

$$\vec{p} = q(2\vec{a})$$

SI unit is C m.

- **Electric Field due to a Dipole:**
  - **On Axial Line:**

$$E_{axial} = \frac{1}{4\pi\epsilon_0}\frac{2p}{r^3}$$

  - **On Equatorial Line:**

$$E_{equatorial} = \frac{1}{4\pi\epsilon_0}\frac{p}{r^3}$$

- **Electric Potential due to a Dipole:**
  - **On Axial Line:**

$$V_{axial} = \frac{1}{4\pi\epsilon_0}\frac{p}{r^2}$$

  - **On Equatorial Line:** $V_{equatorial} = 0$

- **Torque:**

$$\vec{\tau} = \vec{p} \times \vec{E}, \quad \tau = pE\sin\theta$$

- **Potential Energy:**

$$U = -\vec{p} \cdot \vec{E} = -pE\cos\theta$$

Capacitors

$$C = \frac{q}{V}$$

SI unit: farad (F)

1. Parallel Plate Capacitor:

$$C = \frac{\epsilon_0 A}{d}, \quad \text{With dielectric: } C = \frac{K\epsilon_0 A}{d}$$

2. Spherical Capacitor:

- Isolated Sphere:

$$C = 4\pi\epsilon_0 R$$

- Concentric Spheres:

$$C = 4\pi\epsilon_0 \frac{ab}{b-a}$$

3. Cylindrical Capacitor:

$$C = \frac{2\pi\epsilon_0 L}{\ln(b/a)}$$

Combination of Capacitors

- Series: $\frac{1}{C_{eq}} = \sum \frac{1}{C_i}$
- Parallel: $C_{eq} = \sum C_i$

Energy Stored in Capacitor

$$U = \frac{1}{2}CV^2 = \frac{1}{2}qV = \frac{q^2}{2C}, \quad u = \frac{1}{2}\epsilon_0 E^2$$

Dielectrics

- Polar: have permanent dipole (e.g., water).
- Non-polar: no permanent dipole (e.g., O_2, N_2).

$$K = \frac{C_{dielectric}}{C_{vacuum}} = \frac{\epsilon}{\epsilon_0}$$