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Subject: Physics
Weightage: High
Title: Gravitation

GRAVITATION

Newton's Law of Gravitation

The gravitational force between two masses m_1 and m_2 separated by distance r is:

$$F = \frac{Gm_1m_2}{r^2}$$

where $G = 6.67 \times 10^{-11}\ \text{Nm}^2/\text{kg}^2$ is the gravitational constant.

Variation in Acceleration Due to Gravity (g)

(a) With Altitude

At height h above Earth:

$$g' = \frac{GM}{(R+h)^2} = \frac{g}{\left(1+\frac{h}{R}\right)^2}$$

For $h \ll R$:

$$g' \approx g\left(1 - \frac{2h}{R}\right)$$

(b) With Depth

At depth d inside Earth:

$$g' = g\left(1 - \frac{d}{R}\right)$$

(c) With Earth's Shape

Earth's radius is larger at the equator than at poles, so:

$$g_{pole} > g_{equator}$$

(d) With Rotation

At latitude λ:

$$g' = g - R\omega^2\cos^2\lambda$$

Gravitational Potential Energy

Potential energy of mass m at distance r from mass M:

$$U = -\frac{GMm}{r}$$

Gravitational Potential

Potential at distance r from mass M:

$$V = -\frac{GM}{r}$$

Orbital Velocity of a Satellite

$$v_0 = \sqrt{\frac{GM}{r}}$$

Kinetic energy:

$$K = \frac{1}{2}mv_0^2 = \frac{GMm}{2r}$$

Total energy:

$$E = -\frac{GMm}{2r}$$

Escape Velocity

Minimum velocity to escape Earth's gravity:

$$v_e = \sqrt{\frac{2GM}{R}} = \sqrt{2gR} \approx 11.2\ \text{km/s}$$

$$v_{esc} = \sqrt{\frac{2GM}{R}} = \sqrt{2gR} \approx 11.2 \text{ km/s}$$

Kepler's Laws

1. Law of Orbits

Planets move in elliptical orbits with the Sun at one focus.

2. Law of Areas

Line joining planet to Sun sweeps equal areas in equal times.

3. Law of Periods

$$T^2 \propto a^3, \quad T^2 = \left(\frac{4\pi^2}{GM}\right) a^3$$