Name: Chandra Shekhar
College: IIT Gandhinagar
Subject: Physics
Weightage: Regular
Title: Simple Harmonic Motion

SHM

Periodic Motion

Motion that repeats after regular intervals is called periodic. When a particle retraces its path to and fro around an equilibrium position, it's referred to as oscillatory or vibratory motion.

Key Parameters of Harmonic Motion

- **Amplitude (A):** Maximum displacement from mean position.
- **Time Period (T):** Time taken to complete one full oscillation.
- **Frequency (f):** Number of oscillations per second.

$$f = \frac{1}{T}$$

Simple Harmonic Motion (SHM)

Displacement as a function of time is:

$$x(t) = A\sin(\omega t + \phi)$$

The SHM differential equation:

$$\frac{d^2x}{dt^2} + \omega^2 x = 0$$

For a mass-spring system: $\omega = \sqrt{\frac{k}{m}}$

Energy in SHM

- **Kinetic Energy:** $KE = \frac{1}{2}m\omega^2(A^2 - x^2)$
- **Potential Energy:** $PE = \frac{1}{2}m\omega^2 x^2$
- **Total Energy:** Constant: $E = \frac{1}{2}m\omega^2 A^2$

Simple Pendulum

An ideal pendulum executes SHM for small angular displacements.

$$T = 2\pi\sqrt{\frac{L}{g}}$$

Damped Oscillations

Amplitude decays exponentially:

$$x(t) = A_0 e^{-bt/2m}\cos(\omega' t + \phi)$$

Forced Oscillations and Resonance

- Oscillations under an external periodic force.
- At resonance: external frequency = natural frequency ⇒ maximum amplitude.

Waves and Types

- **Transverse:** Particle displacement ⊥ wave direction.
- **Longitudinal:** Particle displacement ∥ wave direction.

Wave Equation and Velocity

$$y(x,t) = A\sin(kx - \omega t + \phi), \quad v = \frac{\omega}{k} = f\lambda$$

Wave Speeds

- On string: $v = \sqrt{\frac{T}{\mu}}$
- In fluid: $v = \sqrt{\frac{B}{\rho}}$
- In solid: $v = \sqrt{\frac{Y}{\rho}}$

Superposition Principle

Resultant displacement = vector sum of individual wave displacements.

Standing Waves

Formed by superposing two waves of same amplitude/frequency traveling in opposite directions.

- **Node:** Zero amplitude.
- **Antinode:** Maximum amplitude.

Standing Sound Waves

Open Pipe:

$$f_1 = \frac{v}{2L}, \quad f_2 = 2f_1, \quad f_3 = 3f_1 \Rightarrow f_n = nf_1$$

Closed Pipe:

$$f_1 = \frac{v}{4L}, \quad f_3 = 3f_1, \quad f_5 = 5f_1 \Rightarrow f_n = (2n-1)f_1$$

Decibel Scale

$$\beta = 10\log_{10}\left(\frac{I}{I_0}\right)$$

Beats

$$f_{\text{beat}} = |f_1 - f_2|$$

Doppler Effect

$$f' = f\left(\frac{v \pm v_O}{v \mp v_S}\right)$$

Use + for motion towards and − for motion away.