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Title: Kinetic Theory of Gases

Kinetic Theory of Gases

Kinetic Pressure

In an ideal gas, the molecules exert pressure on the container walls due to constant collisions. This pressure is mathematically given by:

$$P = \frac{1}{3}\rho v_{rms}^2$$

Where v_{rms} is the root mean square speed of gas molecules and ρ is the density of the gas.

Kinetic Interpretation of Temperature

The temperature of an ideal gas reflects the average translational kinetic energy of its molecules:

$$K_{av} = \frac{1}{2}mv_{rms}^2 = \frac{3}{2}kT$$

Here, k is the Boltzmann constant (1.38×10^{-23} J/K).

Different Speeds of Gas Molecules

Molecular motion in a gas can be described using three characteristic speeds:

a) Root Mean Square Speed (v_{rms})

$$v_{rms} = \sqrt{\frac{v_1^2 + v_2^2 + \cdots + v_N^2}{N}} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3kT}{m}} = \sqrt{\frac{3P}{\rho}}$$

b) Most Probable Speed (v_{mp})

The speed at which the maximum number of molecules move:

$$v_{mp} = \sqrt{\frac{2RT}{M}} = \sqrt{\frac{2kT}{m}}$$

c) Average Speed (v_{avg})

Arithmetic mean of all molecular speeds:

$$v_{avg} = \frac{v_1 + v_2 + \cdots + v_N}{N} = \sqrt{\frac{8RT}{\pi M}} = \sqrt{\frac{8kT}{\pi m}}$$

Relation among speeds:

$$v_{mp} < v_{avg} < v_{rms}$$

Roughly: $v_{mp} : v_{avg} : v_{rms} :: 1 : 1.128 : 1.224$

Degrees of Freedom (f)

Refers to the number of independent ways a molecule can possess energy:

- Monatomic: $f = 3$ (only translational)
- Diatomic: $f = 5$ (3 translational + 2 rotational)
- Triatomic (non-linear): $f = 6$ (3 translational + 3 rotational)
- Triatomic (linear): $f = 5$ (like diatomic)

Law of Equipartition of Energy

Energy is equally shared among all active degrees of freedom:

- Per molecule per degree: $\frac{1}{2}kT$
- Per mole per degree: $\frac{1}{2}RT$
- Total energy per molecule: $\frac{1}{2}fkT$
- Total energy per mole: $\frac{1}{2}fRT$

Mixture of Gases

Useful results for gas mixtures:

- **Total Internal Energy:**
$$U_{mix} = U_1 + U_2 + \dots$$
- **Average Molar Mass:**
$$M_{mix} = \frac{n_1 M_1 + n_2 M_2 + \dots}{n_1 + n_2 + \dots}$$
- **Dalton's Law of Pressures:**
$$P_{mix} = P_1 + P_2 + \dots$$
- **Average C_V and C_P:**
$$(C_V)_{mix} = \frac{n_1 C_{v_1} + n_2 C_{v_2} + \dots}{n_1 + n_2 + \dots}$$
$$(C_P)_{mix} = \frac{n_1 C_{p_1} + n_2 C_{p_2} + \dots}{n_1 + n_2 + \dots}$$
- **Adiabatic Index:**
$$\gamma_{mix} = \frac{(C_P)_{mix}}{(C_V)_{mix}}$$