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Atoms and Nuclei

Rutherford's Alpha Scattering Experiment

In the Geiger-Marsden experiment, alpha particles were directed at a thin gold foil, leading to key insights into atomic structure.

- **Scattering Dependence:**

$$N \propto \sin^{-4}\left(\frac{\theta}{2}\right)$$

where θ is the scattering angle.

- **Minimum Distance of Approach:**

$$d = \frac{Ze^2}{4\pi\epsilon_0(\frac{1}{2}mv_i^2)}$$

Typically around 10^{-14} m, indicating the nucleus is extremely small.

- **Impact Parameter (b):**

$$b = \frac{Ze^2\cot(\theta/2)}{4\pi\epsilon_0(\frac{1}{2}mv_i^2)}$$

For direct collision, $b = 0$.

Rutherford's Atomic Model

Key conclusions from the scattering experiment:

1. Positive charge and mass are centralized in the nucleus.
2. Electrons orbit the nucleus.
3. Most of the atom is empty space.

Drawbacks:

- Could not justify why electrons don't spiral into the nucleus.
- Could not explain the emission spectra of atoms.

Bohr's Atomic Model for Hydrogen

Foundational Assumptions:

1. Electrons revolve in stable, discrete orbits.
2. Angular momentum is quantized:

$$mvr = n\frac{h}{2\pi}, \quad n = 1, 2, 3, \ldots$$

3. Radiation is emitted or absorbed only during transitions between these orbits:

$$\Delta E = h\nu$$

Important Results (for hydrogen-like species):

- Radius of orbit:

$$r_n = 0.529\frac{n^2}{Z}\ \text{Å}$$

- Electron speed:

$$v_n = 2.18 \times 10^6\frac{Z}{n}\ \text{m/s}$$

- Energy of level:

$$E_n = -13.6\frac{Z^2}{n^2}\ \text{eV}$$

Hydrogen Spectral Series

Electrons transitioning between orbits produce emission lines.

- **Lyman:** $n \to 1$, UV region.
- **Balmer:** $n \to 2$, Visible region.
- **Paschen:** $n \to 3$, IR region.
- **Brackett:** $n \to 4$, IR region.
- **Pfund:** $n \to 5$, IR region.

General formula:

$$\frac{1}{\lambda} = R_H Z^2 \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right)$$

where $R_H = 1.097 \times 10^7 \text{ m}^{-1}$.

Matter Waves: de Broglie Relation

$$\lambda = \frac{h}{mv}$$

For electrons accelerated by potential V:

$$\lambda = \frac{1.227}{\sqrt{V}} \text{ nm}$$

Heisenberg Uncertainty Principle

$$\Delta x \cdot \Delta p \geq \frac{h}{4\pi}$$

Also expressed as:

$$\Delta x \cdot m\Delta v \geq \frac{h}{4\pi}$$

Nuclear Basics

- 1 amu $= 1.66 \times 10^{-27}$ kg $= 931.5$ MeV/c^2.
- **Z:** Proton number, **A:** Total nucleons, **N = A - Z**.

Types of Nuclei

- Isotopes: Same Z, different A.
- Isobars: Same A, different Z.
- Isotones: Same N.
- Isomers: Same Z and A, different energy state.
- Mirror Nuclei: $Z_1 = N_2$, $Z_2 = N_1$, A same.

Nuclear Size and Density

- Radius: $R = R_0 A^{1/3}$, where $R_0 = 1.2 \times 10^{-15}$ m.
- Density: $\approx 2.3 \times 10^{17}$ kg/m^3.

Mass Defect and Binding Energy

- $\Delta m = [Zm_p + (A - Z)m_n] - M_{nucleus}$
- $BE = \Delta mc^2 = \Delta m \times 931.5$ MeV
- BE per nucleon $= \frac{BE}{A}$ (max for Fe-56)

Radioactive Decay

$$\frac{dN}{dt} = -\lambda N \to N = N_0 e^{-\lambda t}$$

- Half-life: $T_{1/2} = \frac{0.693}{\lambda}$
- Mean life: $\tau = \frac{1}{\lambda} = 1.44 T_{1/2}$

Units:

- 1 Bq $=$ 1 disintegration/sec

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- 1 Ci $= 3.7 \times 10^{10}$ Bq
- 1 Rd $= 10^{6}$ Bq

Types of Decay

1. **Alpha:** ${}^{A}_{Z}X \rightarrow {}^{A-4}_{Z-2}Y + {}^{4}_{2}He$
2. **Beta-:** ${}^{A}_{Z}X \rightarrow {}^{A}_{Z+1}Y + {}^{0}_{-1}\beta + \bar{\nu}$
3. **Beta+:** ${}^{A}_{Z}X \rightarrow {}^{A}_{Z-1}Y + {}^{0}_{1}\beta + \nu$
4. **Electron Capture:** ${}^{A}_{Z}X + {}_{-1}e^{0} \rightarrow {}^{A}_{Z-1}Y + \nu$
5. **Gamma:** No change in A or Z. Excited nucleus emits photon.

Nuclear Reactions

Fission

Heavy nucleus splits to form lighter ones, releasing energy.

$$ {}^{235}_{92}U + {}^{1}_{0}n \rightarrow {}^{141}_{56}Ba + {}^{92}_{36}Kr + 3{}^{1}_{0}n + Q \approx 200 \text{ MeV} $$

Chain reaction possible.

Fusion

Light nuclei combine at high temperatures.

$$ {}^{2}_{1}H + {}^{3}_{1}H \rightarrow {}^{4}_{2}He + {}^{1}_{0}n + 17.6 \text{ MeV} $$