

Name: Chandra Shekhar  
College: IIT Gandhinagar  
Subject: Physics  
Weightage: Regular  
Title: Atoms & Nuclei

## Atoms and Nuclei

### Rutherford's Alpha Scattering Experiment

In the Geiger-Marsden experiment, alpha particles were directed at a thin gold foil, leading to key insights into atomic structure.

- **Scattering Dependence:**

$$N \propto \sin^{-4} \left( \frac{\theta}{2} \right)$$

where  $\theta$  is the scattering angle.

- **Minimum Distance of Approach:**

$$d = \frac{Ze^2}{4\pi\epsilon_0(\frac{1}{2}mv_i^2)}$$

Typically around  $10^{-14}$  m, indicating the nucleus is extremely small.

- **Impact Parameter (b):**

$$b = \frac{Ze^2 \cot(\theta/2)}{4\pi\epsilon_0(\frac{1}{2}mv_i^2)}$$

For direct collision,  $b = 0$ .

### Rutherford's Atomic Model

Key conclusions from the scattering experiment:

1. Positive charge and mass are centralized in the nucleus.
2. Electrons orbit the nucleus.
3. Most of the atom is empty space.

**Drawbacks:**

- Could not justify why electrons don't spiral into the nucleus.
- Could not explain the emission spectra of atoms.

### Bohr's Atomic Model for Hydrogen

**Foundational Assumptions:**

1. Electrons revolve in stable, discrete orbits.
2. Angular momentum is quantized:

$$mvr = n\frac{h}{2\pi}, \quad n = 1, 2, 3, \dots$$

3. Radiation is emitted or absorbed only during transitions between these orbits:

$$\Delta E = h\nu$$

**Important Results (for hydrogen-like species):**

- Radius of orbit:

$$r_n = 0.529 \frac{n^2}{Z} \text{ \AA}$$

- Electron speed:

$$v_n = 2.18 \times 10^6 \frac{Z}{n} \text{ m/s}$$

- Energy of level:

$$E_n = -13.6 \frac{Z^2}{n^2} \text{ eV}$$

### Hydrogen Spectral Series

Electrons transitioning between orbits produce emission lines.

- **Lyman:**  $n \rightarrow 1$ , UV region.
- **Balmer:**  $n \rightarrow 2$ , Visible region.
- **Paschen:**  $n \rightarrow 3$ , IR region.
- **Brackett:**  $n \rightarrow 4$ , IR region.
- **Pfund:**  $n \rightarrow 5$ , IR region.

General formula:

$$\frac{1}{\lambda} = R_H Z^2 \left( \frac{1}{n_1^2} - \frac{1}{n_2^2} \right)$$

where  $R_H = 1.097 \times 10^7 \text{ m}^{-1}$ .

## Matter Waves: de Broglie Relation

$$\lambda = \frac{h}{mv}$$

For electrons accelerated by potential  $V$ :

$$\lambda = \frac{1.227}{\sqrt{V}} \text{ nm}$$

## Heisenberg Uncertainty Principle

$$\Delta x \cdot \Delta p \geq \frac{h}{4\pi}$$

Also expressed as:

$$\Delta x \cdot m\Delta v \geq \frac{h}{4\pi}$$

## Nuclear Basics

- $1 \text{ amu} = 1.66 \times 10^{-27} \text{ kg} = 931.5 \text{ MeV}/c^2$ .
- **Z:** Proton number, **A:** Total nucleons, **N = A - Z**.

## Types of Nuclei

- Isotopes: Same Z, different A.
- Isobars: Same A, different Z.
- Isotones: Same N.
- Isomers: Same Z and A, different energy state.
- Mirror Nuclei:  $Z_1 = N_2$ ,  $Z_2 = N_1$ , A same.

## Nuclear Size and Density

- Radius:  $R = R_0 A^{1/3}$ , where  $R_0 = 1.2 \times 10^{-15} \text{ m}$ .
- Density:  $\approx 2.3 \times 10^{17} \text{ kg/m}^3$ .

## Mass Defect and Binding Energy

- $\Delta m = [Zm_p + (A - Z)m_n] - M_{\text{nucleus}}$
- $BE = \Delta mc^2 = \Delta m \times 931.5 \text{ MeV}$
- BE per nucleon =  $\frac{BE}{A}$  (max for Fe-56)

## Radioactive Decay

$$\frac{dN}{dt} = -\lambda N \Rightarrow N = N_0 e^{-\lambda t}$$

- Half-life:  $T_{1/2} = \frac{0.693}{\lambda}$
- Mean life:  $\tau = \frac{1}{\lambda} = 1.44 T_{1/2}$

**Units:**

- $1 \text{ Bq} = 1 \text{ disintegration/sec}$

- 1 Ci =  $3.7 \times 10^{10}$  Bq
- 1 Rd =  $10^6$  Bq

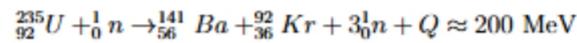
## Types of Decay

1. **Alpha:**  ${}^A_Z X \rightarrow {}^{A-4}_{Z-2} Y + {}^4_2 He$
2. **Beta-:**  ${}^A_Z X \rightarrow {}^A_{Z+1} Y + {}^0_{-1} \beta + \bar{\nu}$
3. **Beta+:**  ${}^A_Z X \rightarrow {}^A_{Z-1} Y + {}^0_1 \beta + \nu$
4. **Electron Capture:**  ${}^A_Z X + {}^0_{-1} e^0 \rightarrow {}^A_{Z-1} Y + \nu$
5. **Gamma:** No change in A or Z. Excited nucleus emits photon.

## Nuclear Reactions

### Fission

Heavy nucleus splits to form lighter ones, releasing energy.



Chain reaction possible.

### Fusion

Light nuclei combine at high temperatures.

