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# Mathematical Reasoning

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## 1. Basic Logical Statements

| Term | Meaning |
|---|---|
| Statement | A sentence that is either true or false, not both |
| Compound Statement | Combination of two or more statements |
| Negation ($\sim p$ or $\neg p$) | "Not $p$", the opposite truth value of $p$ |
| Conjunction ($p \wedge q$) | "$p$ and $q$", true only when both are true |
| Disjunction ($p \vee q$) | "$p$ or $q$", true if at least one is true |
| Implication ($p \rightarrow q$) | "If $p$ then $q$" |
| Converse ($q \rightarrow p$) | Reverses the implication |
| Inverse ($\neg p \rightarrow \neg q$) | Negates both parts of the implication |
| Contrapositive ($\neg q \rightarrow \neg p$) | Negates and reverses |
| Biconditional ($p \leftrightarrow q$) | "$p$ if and only if $q$" |
| Tautology | Always true |
| Contradiction | Always false |

## 2. Truth Tables

| $p$ | $q$ | $p \wedge q$ | $p \vee q$ | $p \rightarrow q$ |
|---|---|---|---|---|
| T | T | T | T | T |
| T | F | F | T | F |
| F | T | F | T | T |
| F | F | F | F | T |

| $p$ | $\sim p$ | $p \leftrightarrow q$ (iff) |
|---|---|---|
| T | F | T if $p = q$; F otherwise |
| F | T | |

## 3. Logical Equivalences

- $\neg(p \wedge q) = \neg p \vee \neg q$     (De Morgan's Law)

- $\neg(p \vee q) = \neg p \wedge \neg q$     (De Morgan's Law)

- $p \rightarrow q \equiv \neg p \vee q$

- $p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$

- Double Negation: $\neg(\neg p) = p$

## 4. Quantifiers

| Quantifier | Meaning |
|---|---|
| Universal ($\forall$) | "For all" |
| Existential ($\exists$) | "There exists" |
| Negation of $\forall x \in S : P(x)$ | $\exists x \in S : \neg P(x)$ |

| | |
|---|---|
| Negation of $\exists x \in S : P(x)$ | $\forall x \in S : \neg P(x)$ |

## 5. Methods of Proof (JEE Context)

[label=—]**Direct Proof:** Use logical implications directly to reach conclusion. **Contrapositive Proof:** Prove $\neg q \rightarrow \neg p$ instead of $p \rightarrow q$. **Proof by Contradiction:** Assume the negation of the statement and derive a contradiction. **Using Truth Tables:** Useful for verifying equivalences and implications.

## 6. Key Concepts to Crack JEE Reasoning Problems

- Always break down compound statements using truth tables or equivalences.
2. Learn implication equivalence: $p \rightarrow q \equiv \neg p \vee q$.
3. Understand contrapositive vs converse. Only contrapositive is logically equivalent.
4. Identify tautology, contradiction, and contingency clearly.
5. De Morgan's Laws are crucial for simplifying negations.
6. Use quantifier negation rules carefully in "for all" and "there exists" questions.
7. For reasoning questions in JEE, identify valid vs invalid argument forms.
8. Apply substitution using equivalences to simplify complex statements.
9. Visual truth tables help eliminate incorrect MCQs quickly.
10. Logical reasoning often appears disguised in functional, set, or algebra problems—watch out!