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 Title: Circle

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Standard Forms of Circle

Equation	Centre	Radius
$x^2 + y^2 = r^2$	$(0, 0)$	r
$(x - a)^2 + (y - b)^2 = r^2$	(a, b)	r
General form: $x^2 + y^2 + 2gx + 2fy + c = 0$	$(-g, -f)$	$\sqrt{g^2 + f^2 - c}$

Basic Properties

Property	Formula / Description
Centre (from general form)	$(-g, -f)$
Radius (from general form)	$r = \sqrt{g^2 + f^2 - c}$
Diameter	$2r$
Area	πr^2
Circumference	$2\pi r$
Chord of length l at distance d from center	$l = 2\sqrt{r^2 - d^2}$

Equation Types

- Chord with midpoint (x_1, y_1) :

$$T = S_1 \Rightarrow xx_1 + yy_1 = x_1^2 + y_1^2 - r^2$$

- Tangent at point (x_1, y_1) on the circle:

$$xx_1 + yy_1 = r^2$$

- Length of Tangent from external point (x_1, y_1) :

$$\sqrt{(x_1 - a)^2 + (y_1 - b)^2 - r^2}$$

- Equation of tangent from external point (x_1, y_1) :

$$S_1 = 0, \text{ solve for pair of tangents and find the line(s)}$$

- Family of circles passing through two points:

$$S_1 + \lambda S_2 = 0$$

Important Loci and Geometry

- Locus of point equidistant from two fixed points: Perpendicular bisector.
- Circle touching a line and passing through a point: Use point-to-line distance = radius.
- Orthogonality of circles: Two circles S_1 and S_2 intersect orthogonally if:

$$2g_1g_2 + 2f_1f_2 = c_1 + c_2$$

- Radical axis of two circles:

$$S_1 - S_2 = 0$$

Key Concepts to Solve JEE Problems on Circle

1. **Use symmetry:** Circle is symmetric about center — exploit in locus/geometry questions.
2. **Convert to standard form:** Always complete the square for general form.
3. **Use $T = S_1$ for chord with given midpoint.**
4. **Tangents from external points:** Use $xx_1 + yy_1 = r^2$ for direct tangent.
5. **Power of a point:** $(\text{Distance})^2 - r^2 = \text{length}^2$ of tangent.
6. **Orthogonal circles:** Check $2g_1g_2 + 2f_1f_2 = c_1 + c_2$.
7. **Family of circles:** Use $S + \lambda S' = 0$ to find required circle.
8. **Locus problems:** Use distance formula and eliminate variables systematically.
9. **Circle touching axes:** If a circle touches coordinate axes, center is at (r, r) or $(r, -r)$ etc.
10. **Use coordinate geometry identities:** Many problems reduce to using circle as geometric constraint.